Testing General Metric Theories of Gravity with Bursting Neutron Stars

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I show that several observable properties of bursting neutron stars in metric theories of gravity can be calculated using only conservation laws, Killing symmetries, and the Einstein equivalence principle, without requiring the validity of the general relativistic field equations. I calculate, in particular, the gravitational redshift of a surface atomic line, the touchdown luminosity of a radius-expansion burst, which is believed to be equal to the Eddington critical luminosity, and the apparent surface area of a neutron star as measured during the cooling tails of bursts. I show that, for a general metric theory of gravity, the apparent surface area of a neutron star depends on the coordinate radius of the stellar surface and on its gravitational redshift in the exact same way as in general relativity. On the other hand, the Eddington critical luminosity depends also on an additional parameter that measures the degree to which the general relativistic field equations are satisfied. These results can be used in conjunction with current and future high-energy observations of bursting neutron stars to test general relativity in the strong-field regime.

I. INTRODUCTION

Black holes and neutron stars probe the strongest gravitational fields found in the present-day universe. Astrophysical systems with properties that are dominated by such compact objects are routinely observed throughout the electromagnetic spectrum and will soon be detected by gravitational wave observatories. It is our hope and expectation that such observations will allow for clean tests of strong-field general relativity and will demonstrate beyond doubt that black-hole candidates are surrounded by event horizons.

Testing strong-field gravity and searching for evidence of event horizons with current and future observations require a theoretical framework within which such observations can be understood and possible violations of the general relativistic predictions can be quantified. Because of the strong gravitational field and the relativistic velocities found in the vicinities of compact objects, the Parametric Post-Newtonian framework [1], which has been very successful in performing weak-field tests, cannot be used.

Current studies are either based on phenomenological spacetimes within the context of general relativity [2], or employ theories that are derived from parametric extensions of the Einstein-Hilbert action [3]. Albeit useful in performing null hypothesis experiments and strong-field tests, these approaches do not necessarily explore the whole range of possible violations of the general relativistic predictions.

In this paper, I show that several observable quantities of bursting neutron stars can be calculated without requiring the validity of Einstein's field equations. Instead, they can be derived from conservation laws and Killing symmetries, assuming only the validity of the Einstein equivalence principle. This alleviates the need for a general parametric theory with which these strong-field tests of gravity can be performed. It also allows for a direct translation of observable quantities such as redshifts, luminosities, temperatures, and apparent surface areas, into values of the metric elements on the surfaces of neutron stars (see §III).

In the calculations reported here, I assume only the validity of the Einstein equivalence principle. This is one of the fundamental building blocks of gravity theories and encompasses the concepts of the equivalence between inertial and gravitational mass, of local Lorentz invariance, and of local position invariance [1]. A violation of the equivalence principle has severe implications for many non-gravitational experiments. This has allowed solar-system and laboratory experiments to place constraints on the amplitudes of such violations that reach as low as one part in 10^{15} [1], justifying largely the assumption that the Einstein equivalence principle is valid even in the strong gravitational fields found in the vicinities of compact objects. Note here that the validity of the Einstein equivalence principle does not imply the validity also of the strong equivalence principle and hence, in a general metric theory, a self gravitating body would not be following the same geodesics as the test particles considered in this study.

Finally, I assume for simplicity that the neutron stars are not rapidly spinning. This allows for the result to be expressed in terms of only three parameters of the metric of each compact object. The calculation can be extended in principle to incorporate effects of rapid rotation. In that case, however, the results will depend on at least two additional metric elements.

In §II, I describe the formalism used in the calculation and justify the relevant assumptions. In §III, I calculate three observables related to the spectra of bursting neutron stars that have been used in the past both in measuring their masses and radii [4] and in constraining scalar-tensor gravity theories [3]. They are: the gravitational redshift of atomic lines from their surfaces, the touchdown luminosities of radius-expansion bursts, and their apparent surface areas during the cooling of the thermonuclear bursts. I conclude in §IV with a discussion of the prospect of using the calculations reported here in performing strong-field tests of general relativity

with X-ray observations of neutron stars.

II. ASSUMPTIONS AND DEFINITIONS

In the following sections, I will calculate a number of observable properties of a bursting neutron star, assuming only the validity of the Einstein equivalence principle [1], without relying on any field equations for the underlying gravity theory. I will assume for simplicity that the compact object is slowly spinning so that its spacetime can be cast in the general form

$$ds^2 = -\mathcal{R}^2 dt^2 + \mathcal{V}^2 dr^2 + r^2 d\Omega^2 \ . \tag{1}$$

I will also assume that the redshift factor, $\mathcal{R}$, and the volume correction factor, $\mathcal{V}$, are only functions of the coordinate radius r with the proper Minkowskian asymptotic limits. In this spacetime, I will denote the coordinate radius of the stellar surface by r_s and the corresponding values of the metric elements there by $\mathcal{R}_\mathrm{s}$ and $\mathcal{V}_\mathrm{s}$. In general relativity, as well as in a number of alternative gravity theories, $\mathcal{R}\mathcal{V} = 1$ in the spacetime external to the star and $\mathcal{R}_\mathrm{s}\mathcal{V}_\mathrm{s} \simeq 1$ in its low-density surface layers.

Because of the assumption of the validity of the equivalence principle, matter and photons follow geodesics of the spacetime described by the metric (1). I will describe the motion of a massive particle in terms of its 4-velocity $u^\mu \equiv (u^t, u^r, u^\theta, u^\phi)$. I will calculate the u^t and u^r components of the 4-velocity using the conservation laws that arise from the two Killing vectors with components $\xi^\mu = (1,0,0,0)$ and $\eta^\mu = (0,0,0,1)$ of the spacetime, i.e., the conservation of energy $E = -\xi^\mu u_\mu = \mathcal{R}^2 u^t$ and of angular momentum $l = \eta^\mu u_\mu = r^2 u^\phi$. Without loss of generality, I will rotate the coordinate system so that the orbit of the particle is at the equatorial plane, i.e., I will set $\sin\theta = 1$ and $u^\theta = 0$. Finally, I will calculate the last component of the 4-velocity of the particle using the requirement $u_\mu u^\mu = -1$, which leads to

$$u^r = \left[\frac{E^2}{\mathcal{R}^2\mathcal{V}^2} - \frac{1}{\mathcal{V}^2}\left(1 + \frac{l^2}{r^2}\right)\right]^{1/2} \ . \tag{2}$$

I will describe the motion of a photon in terms of its 4-momentum $k^\mu = (k^t, k^r, k^\theta, k^\phi)$. Because of the same Killing vectors and symmetries as in the case of massive particles, I can again write $k^t = E/\mathcal{R}^2$, $k^\theta = 0$, and $k^\phi = l/r^2$. Finally, I will calculate the radial component of the photon 4-momentum from the requirement $k^\mu k_\mu = 0$, which gives

$$k^r = \left(\frac{E^2}{\mathcal{R}^2\mathcal{V}^2} - \frac{1}{\mathcal{V}^2}\frac{l^2}{r^2}\right)^{1/2} \ . \tag{3}$$

Because the spacetime is asymptotically Minkowskian, by assumption, the ratio $b \equiv l/E$ is the asymptotic impact parameter of the photon at radial infinity.

In the following, I will also make frequent use of the 4-velocity of a static observer at coordinate radius r, which is given by

$$u^\mu_\mathrm{obs} = \left(\frac{1}{\mathcal{R}}, 0, 0, 0\right) \ . \tag{4}$$

III. OBSERVED PROPERTIES OF NEUTRON STARS

Thermally emitting neutron stars are prime candidates both for constraining the equation of state of neutron-star matter [4] and for testing general relativity in the strong-field regime [3]. This is especially true for neutron stars that experience strong thermonuclear flashes on their surface layers, which are observed as type I X-ray bursts [5].

The three observable properties of bursting neutron stars that can be used in testing strong-field general relativity are the gravitational redshift of a surface atomic line, the touchdown luminosity of a radius-expansion burst, and the apparent surface area during the cooling phases of the bursts. I will now calculate them in detail.

A. Gravitationally Redshifted Lines

If the spectrum of a thermally emitting neutron star has absorption (or emission) features characteristic of atomic transitions, these features will be detected by an observer at infinity with a gravitational redshift equal to

$$z_\mathrm{s} \equiv \frac{\delta\lambda}{\lambda_0} = \mathcal{R}_\mathrm{s}^{-1} - 1 \ , \tag{5}$$

where λ_0 is the rest-frame wavelength of the atomic transition. For a slowly-spinning neutron star, the emission line will be rotationally broadened to a width $\Delta\lambda$ of

$$\Delta\lambda \simeq 2\frac{\Omega r_\mathrm{s}}{c} \ , \tag{6}$$

where Ω is the spin frequency of the star.

B. Touchdown Luminosity of a Radius-Expansion Burst

The brightest among the type I X-ray bursts from an accreting neutron star show strong spectroscopic evidence for rapid expansion of the radius of the X-ray photosphere [7]. It is widely believed that the luminosities of these bursts reach the Eddington critical luminosity at which the outward radiation force balances gravity, causing the expansion of the surface layers of the neutron star. The touchdown luminosities of radius-expansion bursts from a given source remain constant between bursts to within a few percent, giving empirical verification to the theoretical expectation that the emerging luminosity is approximately equal to the Eddington critical luminosity [7].

In general relativity, the Eddington critical luminosity at the surface of bursting neutron star depends on its mass and radius. I calculate here this critical luminosity for a compact object with an external spacetime described by the metric (1), following refs. [8, 9].

I define the Eddington limit as the critical flux at which the outward radiation force keeps the radial velocity of a particle constant, i.e., the one for which

$$\frac{d^2 r}{d\tau^2} = 0 \,, \tag{7}$$

where τ is the proper time for the particle. If f^μ is the 4-force that correspond to the Eddington limit, then from the geodesic equation in the radial direction

$$\frac{1}{m} f^r = \frac{d^2 r}{d\tau^2} + \Gamma^r_{\mu\nu} u^\mu u^\nu = \Gamma^r_{\mu\nu} u^\mu u^\nu \,, \tag{8}$$

where m is the rest mass of the particle and u^μ its 4-velocity. The only non-zero component of the connection that enters the geodesic equation (8) is

$$\Gamma^r_{tt} = -\frac{\mathcal{R}}{\mathcal{V}^2} \frac{\partial \mathcal{R}}{\partial r} \,. \tag{9}$$

For a particle at rest with a 4-velocity $u^\mu = (\mathcal{R}^{-1}, 0, 0, 0)$, the radiation force that corresponds to the Eddington limit becomes

$$\frac{1}{m} f^r = -\frac{1}{\mathcal{R}\mathcal{V}^2} \frac{\partial \mathcal{R}}{\partial r} \,. \tag{10}$$

The outward radiation force is related to the radiation flux in the particle's rest frame F^r via

$$f^r = \sigma F^r \,, \tag{11}$$

where σ is the cross section for interaction. For a particle at rest, the radiation flux is given by

$$F^r = -T^{(r)(t)} u_t = \mathcal{R} T^{(r)(t)} \,, \tag{12}$$

where $T^{(r)(t)}$ is one element of the energy tensor of the radiation field and the parentheses in the indices emphasize the fact that no summation over the repeated index is implied. The rt-element of the energy tensor of the radiation field at any coordinate radius r can be calculated from (ref. [8] eq. (3.32))

$$T^{(\hat{t})(\hat{r})} = \pi I(r) \sin^2 \alpha \,, \tag{13}$$

where the hats on the indices indicate the fact that these are the tetrad components of the tensor. Using the notation of Ref. [8],

$$T^{(t)(r)} = \frac{1}{\mathcal{R}\mathcal{V}} T^{(\hat{t})(\hat{r})} = \frac{\pi I(r) \sin^2 \alpha}{\mathcal{R}\mathcal{V}} \,. \tag{14}$$

Here α is the opening angle of the star as viewed by a static observer at coordinate radius r, and $I(r_{\rm s})$ is the frequency-integrated specific intensity of the radiation, which I consider here to be isotropic within the opening angle α.

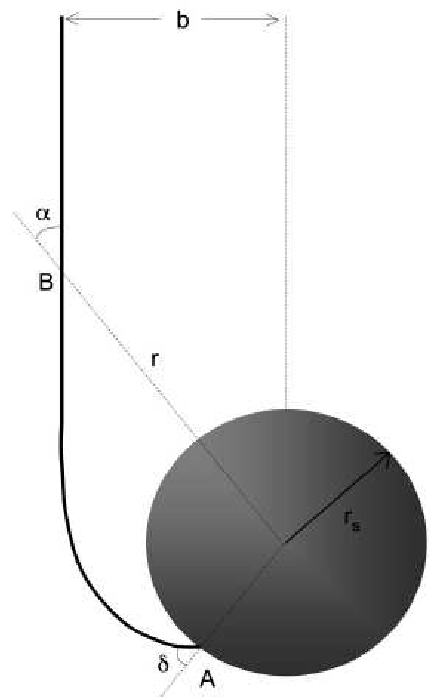

FIG. 1: The geometry used in deriving the Eddington critical luminosity for a slowly spinning neutron star.

In order to calculate the opening angle α, I will follow the procedure of ref. [10], as illustrated in Figure 1. Let point B be the position of a static observer in the spacetime with a 4-velocity u^μ (eq. (4)), and also of two photons with 4-momenta w^μ and v^μ. The angle ξ between the two photons as measured by the static observer is

$$\cos \xi = 1 + \frac{v^\mu w^\mu}{(u^\nu w_\nu)(u^\sigma v_\sigma)} \,. \tag{15}$$

I will consider one photon to be traveling radially outwards, i.e., with a 4-momentum vector that corresponds to $l = 0$ (eq. 3))

$$w^\mu = \left(\frac{\mathcal{V}}{\mathcal{R}}, 1, 0, 0 \right) \tag{16}$$

and the other to be is traveling on a non-radial null geodesic

$$v^\mu = E \left(\frac{1}{\mathcal{R}^2}, \left[\frac{1}{\mathcal{R}^2 \mathcal{V}^2} - \frac{b^2}{\mathcal{V}^2 r^2} \right]^{1/2}, 0, \frac{b}{r^2} \right) \,, \tag{17}$$

where $b \equiv l/E$. The angle between the two photons as measured by the static observer is

$$\cos \xi = \left(1 - \frac{\mathcal{R}^2}{r^2} b^2 \right)^{1/2} \,, \tag{18}$$

from which it follows that

$$\frac{r}{\mathcal{R}}\sin\xi = b \tag{19}$$

is a constant of motion for each photon. This allows me to calculate the opening angle of the star as viewed by an observer at coordinate radius r.

A photon that leaves the stellar surface at point A at an angle δ with respect to the radial direction is detected by the observer at radius r traveling at an angle α with respect to the local radial direction. In its motion, the photon follows a null geodesic characterized by a single impact parameter b and hence

$$b = \frac{r_s}{\mathcal{R}_\mathrm{s}}\sin\delta = \frac{r}{\mathcal{R}}\sin\alpha\ . \tag{20}$$

If the stellar surface is at a coordinate radius larger than the one of the photon orbit, then the photon with the largest angle α detected by the observer is the one that emerged from the stellar surface at $\delta = \pi/2$. If the stellar surface is inside the photon orbit then the opening angle corresponds to $\delta < \pi/2$. As a result, the opening angle of the source as seen by the static observer is

$$\sin\alpha \le \frac{\mathcal{R}}{\mathcal{R}_\mathrm{s}}\frac{r_\mathrm{s}}{r}\ , \tag{21}$$

with the equal sign corresponding to stars larger than the photon orbit radius. Because I will be looking for small deviations from general relativistic neutron stars, all of which are larger than the radius of the photon orbit, I will consider here only this case. It is interesting that, although the details of self-lensing depend on the proper volume factor $\mathcal{V}$, the opening angle of the source as seen by an observer at infinity depends only on the redshift factor $\mathcal{R}$, as in general relativity.

The radiation force on a particle becomes

$$f^r = \frac{\sigma\mathcal{R}\pi I(r)}{\mathcal{R}\mathcal{V}}\left(\frac{\mathcal{R}}{\mathcal{R}_\mathrm{s}}\right)^2\left(\frac{r_\mathrm{s}}{r}\right)^2\ . \tag{22}$$

In order for the particle to remain at a constant radius r, this force has to be equal to (see eq. (10))

$$\frac{1}{\mathcal{R}\mathcal{V}}\frac{\sigma}{m}\mathcal{R}\pi I(r)\left(\frac{\mathcal{R}}{\mathcal{R}_\mathrm{s}}\right)^2\left(\frac{r_\mathrm{s}}{r}\right)^2 = -\frac{1}{\mathcal{R}\mathcal{V}^2}\frac{\partial\mathcal{R}}{\partial r}\ . \tag{23}$$

Evaluating this expression at the stellar surface, I obtain

$$\frac{\sigma}{m}\mathcal{R}_\mathrm{s}^2\pi I(r_\mathrm{s}) = -\frac{\mathcal{R}}{\mathcal{V}}\left.\frac{\partial\mathcal{R}}{\partial r}\right|_{r_\mathrm{s}}\ . \tag{24}$$

I now connect the specific intensity of the radiation field at the stellar radius with that at infinity. Conservation of the photon occupation number

$$n \equiv \frac{I(r_\mathrm{s})}{g_{\mu\nu}k^\mu u^\nu_\mathrm{obs}} \tag{25}$$

leads to

$$I(r\rightarrow\infty) = I(r_\mathrm{s})\mathcal{R}_\mathrm{s}^4\ , \tag{26}$$

from which I can also infer for the luminosity $L_\mathrm{s} = 4\pi r_\mathrm{s}^2\pi I(r_\mathrm{s})$ that

$$L(r\rightarrow\infty) = L(r_\mathrm{s})\mathcal{R}_\mathrm{s}^2\ . \tag{27}$$

As a result, condition (23) becomes

$$L_\mathrm{E}^\infty = \frac{4\pi m r_\mathrm{s}^2}{\sigma}\left(-\frac{\mathcal{R}_\mathrm{s}}{\mathcal{V}_\mathrm{s}}\left.\frac{\partial\mathcal{R}}{\partial r}\right|_{r_\mathrm{s}}\right) = \frac{4\pi m}{\sigma}\frac{r_\mathrm{s}^2 g_\mathrm{eff}}{(1+z_\mathrm{s})^2}\ , \tag{28}$$

where I have defined L_E^∞ as the Eddington critical luminosity as measured by an observer at infinity and the effective acceleration at the stellar surface by

$$g_\mathrm{eff} = \frac{1}{\mathcal{R}_\mathrm{s}\mathcal{V}_\mathrm{s}}\left.\frac{d\mathcal{R}}{dr}\right|_{r_\mathrm{s}}\ . \tag{29}$$

C. Apparent Surface Area During Burst Cooling

A typical type I X-ray burst on a neutron star is characterized by a rapid ($\simeq$ 1 s) rise and a slower (tens of seconds) decay, during which the thermonuclear flash has spread throughout the entire neutron star. Observations of the cooling tails of multiple type I bursts from a single source have shown that the apparent surface area of the emitting region, defined as

$$S^\infty \equiv \frac{4\pi D^2 F_{\mathrm{c},\infty}}{\sigma_\mathrm{SB}T_{\mathrm{c},\infty}^4} \tag{30}$$

remains approximately constant during each burst and between bursts from the same source. In definition (30), $F_{\mathrm{c},\infty}$ is the measured flux of the source during the cooling tail of the burst, T_c^∞ is the measured color temperature of the burst spectrum, D is the distance to the source and σ_SB is the Stefan-Boltzmann constant. In general relativity, the apparent surface area during the cooling tails of X-ray bursts from a neutron star depends on the stellar mass and radius. I calculate here this same quantity for an object described by a general spherically symmetric metric.

The color temperature on the surface of the neutron star is related to the color temperature measured at infinity by a simple redshift, i.e.,

$$T_{\mathrm{c,s}} = \frac{T_{\mathrm{c},\infty}}{\mathcal{R}_\mathrm{s}}\ . \tag{31}$$

I can then use equations (12), (13), and (21) to calculate the radiation flux measured at infinity as

$$F_{\mathrm{c},\infty} = \pi I(r_\mathrm{s})\mathcal{R}_\mathrm{s}^2\left(\frac{r_\mathrm{s}}{D}\right)^2\ . \tag{32}$$

If the spectrum of a neutron star during the cooling tail of a burst were a pure blackbody, then $\pi I(r_\mathrm{s}) =$

$\sigma_{\rm SB}[T(r_{\rm s})]^4$. However, the interaction of photons with the atmospheric electrons and atoms distort the shape of the emerging spectrum. The distortion is typically parameterized in terms of a color correction factor

$$f_{\rm c} \equiv \frac{T_{\rm c}}{T_{\rm eff}}\;, \tag{33}$$

where $T_{\rm c}$ and $T_{\rm eff}$ are the color temperature and the effective temperatures of the atmosphere, respectively. The value of the color correction factor can be calculated a priori and depends on the temperature and the composition of the atmosphere as well as on the local acceleration [11]. With this definition,

$$\pi I(r_{\rm s}) = \frac{\sigma_{\rm SB} T_{\rm c,s}^4}{f_{\rm c}^4} \tag{34}$$

and the observed ratio $F_{\rm c,\infty}/\sigma_{\rm SB}T_{\rm c,\infty}^4$ becomes

$$\frac{F_{\rm c,\infty}}{\sigma_{\rm SB}T_{\rm c,\infty}^4} = \frac{1}{f_{\rm c}^4 \mathcal{R}_{\rm s}^2}\frac{r_{\rm s}^2}{D^2} = \frac{1}{f_{\rm c}^4}\left(\frac{r_{\rm s}}{D}\right)^2 (z_{\rm s}+1)^2\;. \tag{35}$$

It is worth emphasizing that the dependence of the observable on the coordinate radius of the stellar surface and on the redshift is the same as in the case of general relativity.

IV. TESTING GENERAL RELATIVITY WITH BURSTING NEUTRON STARS

In §III, I showed that various observables from a slowly spinning neutron star depend on three parameters related to the coordinate radius of its surface and the values of the metric elements there. In general relativity, however, all these observables depend on only two parameters: the gravitational mass of the compact object and the coordinate radius of its stellar surface. The difference in the number of free parameters between a general metric theory and general relativity makes a direct test of the latter theory possible. Indeed, the three parameters of a compact object in a general metric theory can be measured (or at least constrained) using high-energy observations. If general relativity describes accurately gravitational phenomena in the strong-field regime, then the three parameters cannot be independent but they must satisfy a consistency relation, which I calculate below. The degree to which consistency can be demonstrated will provide a measure of the degree to which general relativistic predictions have been tested.

In general relativity, the two components of the metric (1) at the stellar surface are related to the mass M and the radius $r_{\rm s}$ of the compact object by

$$\mathcal{R}_{\rm s} = \mathcal{V}_{\rm s}^{-1} = \left(1-\frac{2M}{r_{\rm s}}\right)\;. \tag{36}$$

Instead of using the mass of the compact object as the second independent variable, I will choose the redshift from its surface,

$$z_{\rm s,GR} = \left(1-\frac{2M}{r_{\rm s}}\right)^{-1} - 1\;. \tag{37}$$

The effective gravitational acceleration in the surface layers of a general relativistic star then becomes

$$g_{\rm eff,GR} = \frac{1}{2r_{\rm s}}\left[\frac{z_{\rm s}(z_{\rm s}+2)}{z_{\rm s}+1}\right]\;. \tag{38}$$

I can now define a parameter η to measure deviations from general relativity by

$$g_{\rm eff} = \eta g_{\rm eff,GR} = \eta\frac{1}{2r_{\rm s}}\left[\frac{z_{\rm s}(z_{\rm s}+2)}{z_{\rm s}+1}\right]\;. \tag{39}$$

This parameter quantifies the degree to which the gravitational acceleration on the surface of a star can be determined entirely by its mass and radius, as predicted by general relativity.

With the above definitions, I can write the three observable quantities for a bursting neutron star of known distance, D, as

$$\frac{\delta\lambda}{\lambda_0} \;=\; z_{\rm s} \tag{40}$$

$$L_{\rm E}^\infty \;\equiv\; 4\pi D^2 F_{\rm E}^\infty = \frac{4\pi m_{\rm p} r_{\rm s}}{(1+X)\sigma_{\rm T}}\left[\frac{z_{\rm s}(z_{\rm s}+2)}{(1+z_{\rm s})^3}\right]\eta \tag{41}$$

$$R_{\rm app} \;\equiv\; D\left(\frac{F_{\rm c,\infty}}{\sigma_{\rm SB}T_{\rm c,\infty}^4}\right)^{1/2} = \frac{r_{\rm s}}{f_{\rm c}^2}(z_{\rm s}+1)\;, \tag{42}$$

where I have introduced the apparent source radius $R_{\rm app}$. I have also assumed that the interaction between the electrons and the photons is due to coherent Thomson scattering and denoted the hydrogen mass fraction of the accreted material by X.

It is important to note that, for a general metric theory of gravity, the first and the third observables depend on the coordinate radius of the neutron-star surface, $r_{\rm s}$, and on its gravitational redshift, $z_{\rm s}$, in the exact same way as in general relativity. On the other hand, the Eddington luminosity depends also on the parameter η, which measures possible deviations from general relativity. Therefore, it is only through measurements of the touchdown luminosity of radius expansion bursts that general relativity can be tested with bursting neutron stars.

In general relativity, $\eta_{\rm GR}=1$, by definition. On the other hand, in other gravity theories, this parameter depends also on the additional gravitational degrees of freedom. For example, in a scalar-tensor gravity as the one described in ref. [3]), η depends on the magnitude of the scalar field at the surface of the neutron star, which in turn depends on the mass distribution inside the star and its coupling to the scalar field.

Not all combinations of values of the three observables are possible in general relativity. Buchdahl's theorem states that no spherical, general relativistic star can exist with a mass-to-radius ratio larger than $M/R > 4/9$.

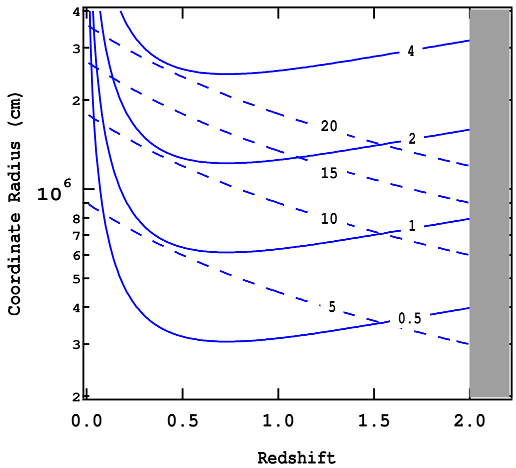

FIG. 2: *Solid lines:* Contours of constant Eddington luminosity at infinity, in units of 10^{38} erg s^{-1}, as a function of the coordinate radius and the redshift of the surface of the bursting neutron star *Dashed lines:* Contours of constant apparent radius of the neutron star, in units of 1 km, as inferred from the cooling tail of a burst, on the same parameter space. In this calculation the atmosphere of the neutron star was assumed to consist of pure hydrogen and the color correction factor was set to 1.34.

This corresponds to a maximum possible value of the redshift from the surface of a general relativistic star of $z_{\rm s}\le 2$. The apparent radius during the cooling tails of bursts depends monotonically on the redshift and, therefore, reaches the highest possible value in general relativity at $z_{\rm s}=2$. Finally, the Eddington luminosity is a non-monotonic function of redshift and peaks at $z_{\rm s}=\sqrt{3}-1\simeq 0.73$. The maximum possible values of the three observables for a general relativistic star are, therefore,

$$\left.\frac{\delta\lambda}{\lambda_0}\right|_{\rm GR} \le 2 \tag{43}$$

$$L_{\rm E,GR}^{\infty} \le (\sqrt{3}-1)\frac{4\pi m r_{\rm s}}{(1+X)\sigma_{\rm T}} \tag{44}$$

$$R_{\rm app,GR} \le \frac{3}{f_{\rm c}^2} r_{\rm s}\;. \tag{45}$$

Violation of any of the above inequalities will also signify that general relativity does not describe accurately gravitational phenomena in the strong-field regime.

A second test of general relativity can be obtained using a slowly-spinning, bursting neutron star at a known distance that accretes matter of known composition and shows radius-expansion type I X-ray bursts. Assuming the validity of general relativity, equations (41) and (42) can be inverted to yield the coordinate radius of the neutron-star surface and its gravitational redshift (see Fig. 2). However, because the Eddington luminosity does not have a monotonic dependence on redshift, this inversion procedure is not always possible. For example, Figure 2 shows that an apparent radius of 5 km and an Eddington luminosity of 2×10^{38} erg s^{-1} are impossible in general relativity.

In order to calculate the range of simultaneously determined values of the Eddington luminosity and of the apparent radius that are consistent with general relativity, I solve equation (41) for r_s and substitute it into equation (42). After a small rearrangement of terms, the result is

$$\frac{(1+X)\sigma_{\rm T}L_{\rm E,GR}^{\infty}}{4\pi m_{\rm p} f_{\rm c}^2 R_{\rm app,GR}} = \frac{z_{\rm s}(z_{\rm s}+2)}{(z_{\rm s}+1)^4}\;. \tag{46}$$

The right-hand-side of this last equation has a maximum value of 1/4 at $z_{\rm s}=\sqrt{2}-1$, so that

$$L_{\rm E,GR}^{\infty} \le \frac{\pi m_{\rm p} f_{\rm c}^2}{(1+X)\sigma_{\rm T}} R_{\rm app,GR}\;, \tag{47}$$

which I can rewrite in CGS units as

$$L_{\rm E,GR}^{\infty} \le 1.9\times10^{38}\left(\frac{f_{\rm c}}{1.34}\right)^2\left(\frac{2}{1+X}\right)\left(\frac{R_{\rm app,GR}}{10^6~{\rm cm}}\right)~{\rm erg~s}^{-1}\;. \tag{48}$$

Violation of inequality (48) will also be evidence for new gravitational physics, not described by general relativity.

The optimal test of general relativity will occur from the detection of gravitationally redshifted lines from a neutron star that is slowly spinning, lies at a known distance, and shows radius-expansion type I X-ray bursts. In this case, equations (40)-(42) can be combined to give

$$\eta = \frac{(1+X)\sigma}{m f_{\rm c}^2} F_{\rm E}^{\infty}\left(\frac{F_{\rm c,\infty}}{\sigma T_{\rm c,\infty}^4}\right)^{-1/2}\frac{(1+\delta\lambda/\lambda_0)^4}{(2+\delta\lambda/\lambda_0)(\delta\lambda/\lambda_0)} D\;. \tag{49}$$

All the quantities on the right-hand side of this last equation are either observable or can be calculated from models of neutron-star atmospheres. The parameter η is a measure of the degree to which general relativity accurately describes the strong gravitational fields found in the vicinities of neutron stars.

A successful application of the line of arguments discussed here relies on the detection of radius-expansion bursts and of gravitationally redshifted lines from sources with known distances. Radius expansions bursts have been detected to date from at least 31 bursting neutron stars [12], several of which lie in globular clusters and, therefore, have also well constrained distances [7]. The touchdown luminosities and apparent radii can be inferred from current data to an accuracy of $\simeq 5\%$ that is limited by systematic uncertainties only [7]. The distances to the globular clusters that contain known bursters are known to an accuracy of $\simeq 5-20\%$. Assuming the latter accuracy to be 10%, and assuming also that the composition of the accreting material can be estimated to the same accuracy by long-wavelength observations of the accretion flow, the proposed test can lead to a measurement of the parameter η to within $\simeq 15\%$.

Note, however, that gravitationally redshifted atomic lines have been reported only for one, slowly rotating, burster, EXO 0748−676. Future observations of bursting neutron stars with telescopes that combine a high

spectral resolution and a large collecting area, such as Constellation-X and XEUS, will allow for quantitative tests of general relativity in the strong-field regime with bursting neutron stars.

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